Measurements of Unloaded Q-Factor of Transmission Mode Dielectric Resonators.

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ABSTRACT

A procedure to determine accurately the unloaded Q-factor, resonant frequency and coupling coefficients of a dielectric resonator in the transmission mode has been developed. The technique is based on a derived equation relating S_{21} and Q_L, measurements of S_{21}, S_{11} and S_{22} and accounts for crosstalk, coupling and cable losses.

1. INTRODUCTION

Measurements of microwave properties of materials using the dielectric resonator [1] require accurate values of the unloaded quality factor Q_o of the resonator. To obtain Q_o, the loaded Q-factor Q_L, resonant frequency f_{res} and coupling coefficients need to be obtained from measurements of S parameters around the resonance.

A measurement environment introduces discrepancies between theoretical dependencies of S_{21}, S_{11} and S_{22} as a function of frequency [2,3] and observed resonance curves. This is due to noise, losses in cables, coupling mechanism and crosstalk and should be accounted for to accurately determine the Q_o-factor. A very useful procedure accounting for these effects was developed for dielectric resonators working in the reflection mode by Kajfez [2,4]. The technique was successfully applied to strongly coupled resonators with Q-factors greater than 100.

So far no similar systematic analysis of dielectric resonators working in the transmission mode has been carried out and this paper attempts to fill the gap. There is a need for such analysis especially as transmission dielectric resonators have become widely used for microwave characterisation of High Temperature Superconductors [5,6,7]. The transmission mode is used instead of the reflection mode for this purpose as S_{21} coefficients exhibit much higher signal to noise ratio than S_{11} or S_{22} for a case of weak coupling (illustrated in Fig. 1 and Fig. 2), and weak coupling prevents EM fields disturbances.

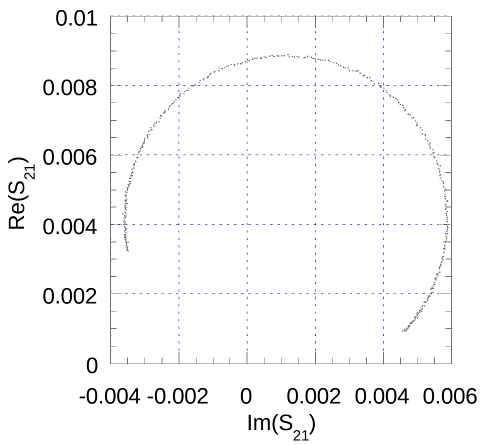

Fig. 1 An experimental S_{21} circle under weak coupling.

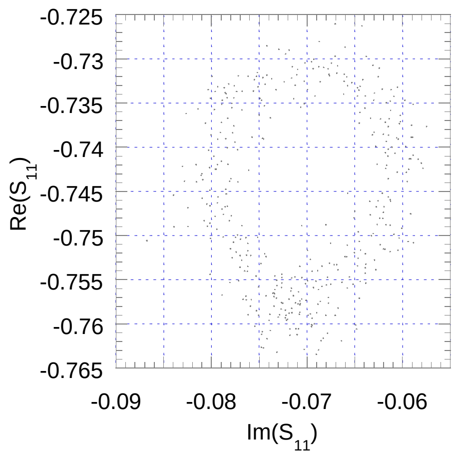

Fig. 2 An experimental S_{11} circle under the same weak coupling condition as in Fig. 1.

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Usually in case of a very low coupling, the Q_o-factor is assumed to be equal to Q_L, what is well justified. To calculate Q_o in case of a low coupling (but not very low to achieve sufficient SNR) the following equation has been typically used.

$$Q_o = \frac{Q_L}{1-|S_{21}|} \qquad (1)$$

This expression is valid in case of equal coupling to the resonator. We have investigated Q_o-factors versus coupling for a 24 GHz sapphire resonator and found that even with coupling loops at equal distances and outside the cavity the assumption of weak coupling and symmetry of coupling is not always fulfilled. This is illustrated in Fig. 3 which presents Q_o-factors calculated using (1) for various positions of the coupling loops and erroneous decrease in Q_o is clearly visible.

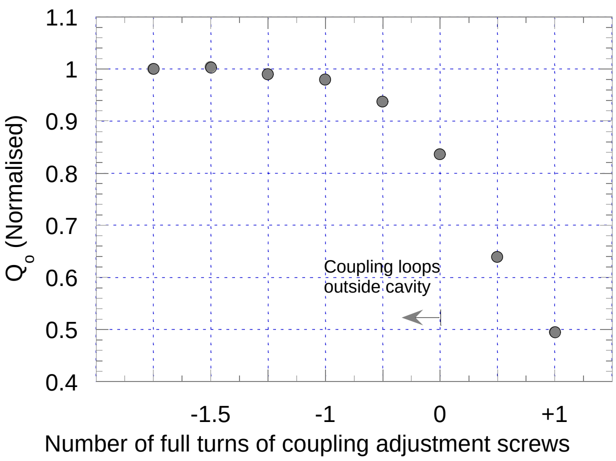

Fig. 3 Inconsistency in Q_o calculated using eqn. (1) over a range of coupling loop positions with both loops at equal distances from the cavity wall.

Hence to obtain accurate values of the Q_o factor, it is evident that the equation, namely

$$Q_o = Q_L(1+\beta_1+\beta_2) \qquad (2)$$

where β_1 and β_2 are the coupling coefficients, needs to be used.

In this paper, we present a method based on a derived equation for a transmission mode dielectric resonator and fractional linear curve fitting to determine the Q_L-factor, f_{res} and β_1 and β_2 from independent measurements of S_{21}, S_{11} and S_{22}. The proposed procedure enables accurate determination of Q_o using (2) in the presence of losses and crosstalk. The method has been verified for a sapphire resonator with copper and superconducting plates. We also compare results obtained using our method with the reflection method of Kajfez over a wide range of coupling. The developed procedure can be directly applied to measurements of surface resistance of High Temperature Superconducting films, and loss tangent measurements of dielectric materials.

2. PROCEDURE TO DETERMINE Q_o-FACTOR FOR TRANSMISSION MODE DIELECTRIC RESONATORS

The well known frequency dependence of the transmission coefficient for a transmission mode resonator is [8,9]:

$$S_{21}(\omega) = \frac{|S_{21}|}{1+j2Q_L(\omega-\omega_o)/\omega_o} \qquad (3)$$

$$|S_{21}| = \frac{2\sqrt{\beta_1}\sqrt{\beta_2}}{1+\beta_1+\beta_2} \qquad (4)$$

Around the resonant angular frequency ω_o, the path traced by the S_{21} vector in the complex plane is a perfect circle which passes through the origin with a centre on the real axis (Fig. 4). In practice a phase shift due to transmission lines connected to the resonator causes the circle to be rotated about the origin. Crosstalk between the coupling loops offsets the circle away from the origin. Losses in the connectors, cables, and coupling mechanism reduces the diameter of the circle. And noise causes roughness in the circle (Fig. 5).

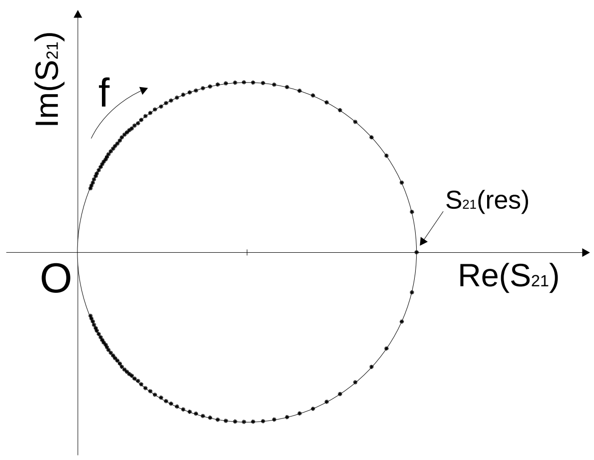

Fig. 4 Ideal S_{21} Q-circle

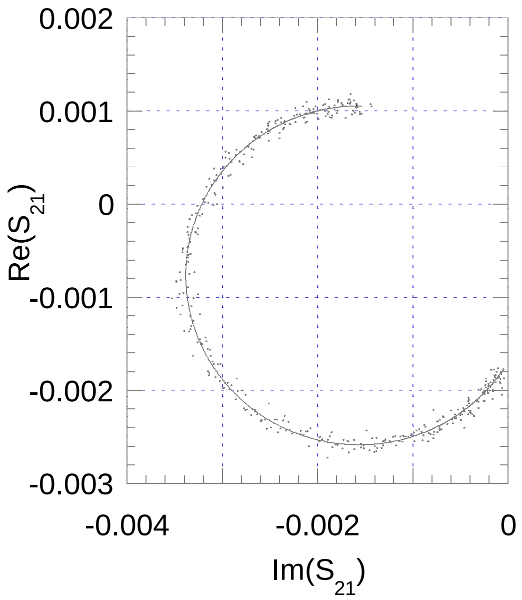

Fig. 5 Practical S_{21} Q-circle

To account for these external factors and to derive a general relationship between Q_L and S_{21} we have used a circuit model of Fig. 6 of a dielectric resonator system, where coupling loss is modelled by a series resistance R_s, and coupling reactance is represented by X_s. The sapphire resonator used subsequently to verify the procedure is shown in Fig. 7.

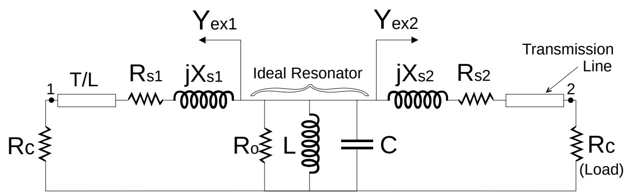

Fig. 6 Resonator circuit model

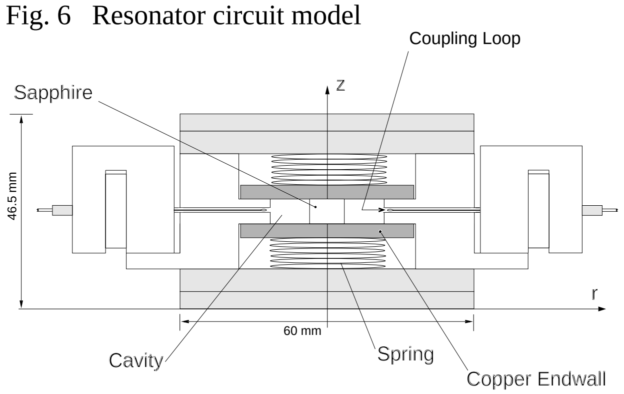

Fig. 7 10 GHz sapphire resonator

Using the following relations for the measurement system model of Fig. 6,

$$G_o = \frac{1}{R_o}; \qquad k = \frac{G_{ex1} + G_{ex2}}{G_o};$$

$$G_{ex1} = Re(Y_{ex1}); \qquad G_{ex2} = Re(Y_{ex2})$$

$$Y_{ex1} = \frac{(R_c + R_{s1}) - jX_{s1}}{(R_c + R_{s1})^2 + X_{s1}^2}; \qquad Y_{ex2} = \frac{(R_c + R_{s2}) - jX_{s2}}{(R_c + R_{s2})^2 + X_{s2}^2} \qquad (5)$$

the transmission coefficient S_{21r} for the transmission mode resonator has been derived as:

$$S_{21r} = \frac{2R_c Y_{ex1} Y_{ex2}}{G_o(1+k)\left[1 + j2Q_L \frac{(\omega - \omega_L)}{\omega_o}\right]} \qquad (6)$$

The functional form of (6) can be presented as

$$S_{21r} = \frac{a_o}{a_3 t + 1} \qquad (7)$$

where
$$t = \frac{2(\omega - \omega_L)}{\omega_o} \qquad (8)$$

and
$$a_3 = jQ_L \qquad (9)$$

Assuming an effect of crosstalk as an addition of a extra complex value to the S_{21} response, the overall response S_{21} may be given by :

$$S_{21} = (S_{21r} + X)e^{j\phi} \qquad (10)$$

where X is a complex constant, and ϕ is the resultant phase shift due to cables, and can be expressed in the following form:

$$S_{21} = \frac{a_1 t + a_2}{a_3 t + 1} \qquad (11)$$

For N values of S_{21} measured around the resonance, a linear fractional linear curve fitting technique have been applied using MATLAB [10] to obtain the complex constants a_1, a_2 and a_3. The loaded Q_L factor is then found by:

$$Q_L = \mathrm{Im}[a_3] \qquad (12)$$

The coupling coefficients necessary to obtain Q_o from (2) are calculated from measured S_{11} and S_{22} at the resonant frequency according to [11]:

$$\beta_1 = \frac{1 - |S_{11}|}{|S_{11}| + |S_{22}|} \qquad (13)$$

$$\beta_2 = \frac{1-|S_{22}|}{|S_{11}|+|S_{22}|} \qquad (14)$$

As reflection responses around the resonance have the same functional form as S_{21}, the same fitting procedure is used to fit circles to experimental S_{11} and S_{22} data. The magnitude of the reflection coefficient of the resonator at f_{res} with coupling losses removed is obtained as:

$$|\Gamma| = \left|\frac{a_2}{(a_1/a_3)}\right| \qquad (15)$$

where $|\Gamma|$ is $|S_{11}|$ or $|S_{22}|$, and a_2 and a_1/a_3 correspond to the resonant and detuned values of the input reflection coefficient.

3. RESULTS

To verify the fitting procedure we have investigated the 10 GHz sapphire resonator with copper plates. Fig. 8 shows Q_o-factors determined for the transmission mode according to the presented procedure and for the reflection mode using the Kajfez procedure [2,4] over a range of coupling loop positions. Obtained Q_o factor

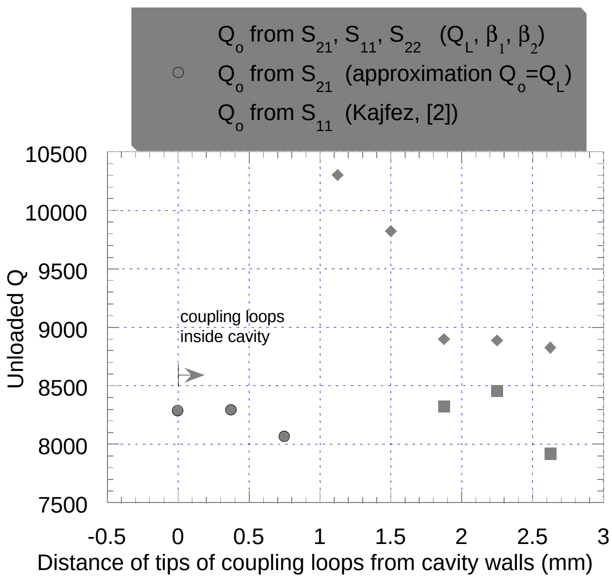

Fig. 8 Unloaded Q Factors measured over a range of coupling loops positions using different procedures.

for the transmission mode for coupling loops at 1.9 mm and 2.25 mm inside the cavity are approximately the same as Q_o for the 0 mm position. The Q_o values obtained for the reflection mode are significantly higher than the unloaded Q_o factor obtained from transmission measurements in the case of low coupling.

4. CONCLUSIONS

For measurements of material properties involving dielectric resonators, weak coupling should be used to prevent the coupling mechanism from disturbing the nominal field distribution in the resonator. Under such weak coupling conditions, it seems that transmission mode measurements S_{21} complimented with reflection mode measurements (S_{11} and S_{22}) if possible, provide better accuracy in the unloaded Q factor than using reflection data alone. In this paper, we have demonstrated a useful procedure for obtaining accurately the Q_o factor of transmission mode resonators.

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